for all right triangles

$$\frac{x}{h} = \cos(\theta)$$

$$\frac{y}{h} = \sin(\theta)$$

The Unit Circle → h = 1

$$x_1 = \cos(\theta)$$
$$y_1 = \sin(\theta)$$

y

$(0,1)$

$(-1,0)$

h

y_1

a

$(1,0)$

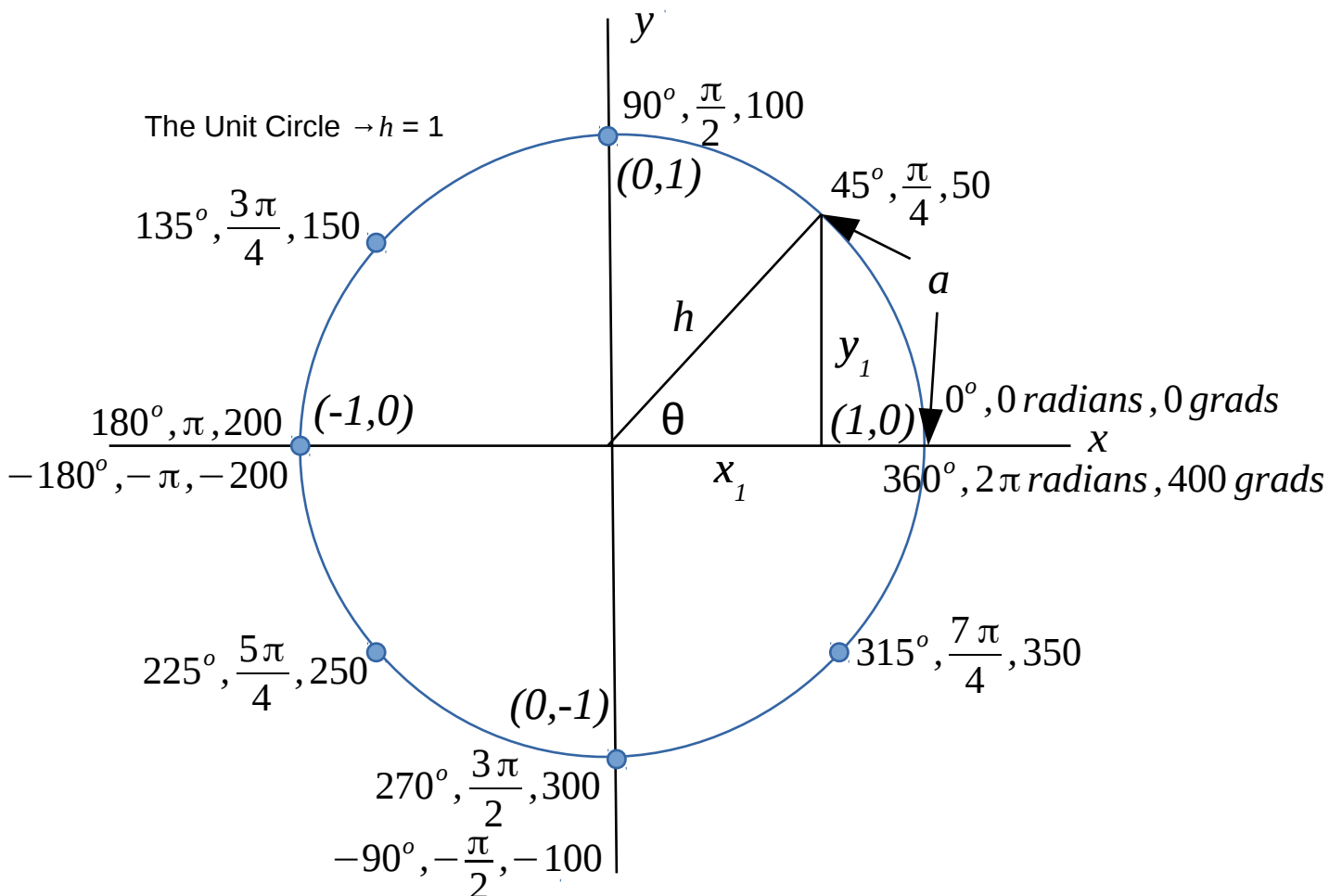θ

x_1

x

$(0,-1)$

for all right triangles

$$\frac{x}{h} = \cos(\theta)$$

$$\frac{y}{h} = \sin(\theta)$$

θ can be measured by degrees (0° to 360°) radians (0 to 2π) grads (0 to 400)

the circumference of the circle is 2πh and the length of an arc is 2πh(θ/2π) = θ since h=1. Note: h = the radius

The Unit Circle → h = 1

$90^o, \frac{\pi}{2}, 100$

(0,1)

$45^o, \frac{\pi}{4}, 50$

$135^o, \frac{3\pi}{4}, 150$

a

h

y_1

$180^o, \pi, 200$ (-1,0)

$-180^o, -\pi, -200$

θ

(1,0)

x_1

$0^o, 0\,radians, 0\,grads$

$360^o, 2\pi\,radians, 400\,grads$

$225^o, \frac{5\pi}{4}, 250$

$315^o, \frac{7\pi}{4}, 350$

(0,-1)

$270^o, \frac{3\pi}{2}, 300$

$-90^o, -\frac{\pi}{2}, -100$

Example: What is cos(30°)?

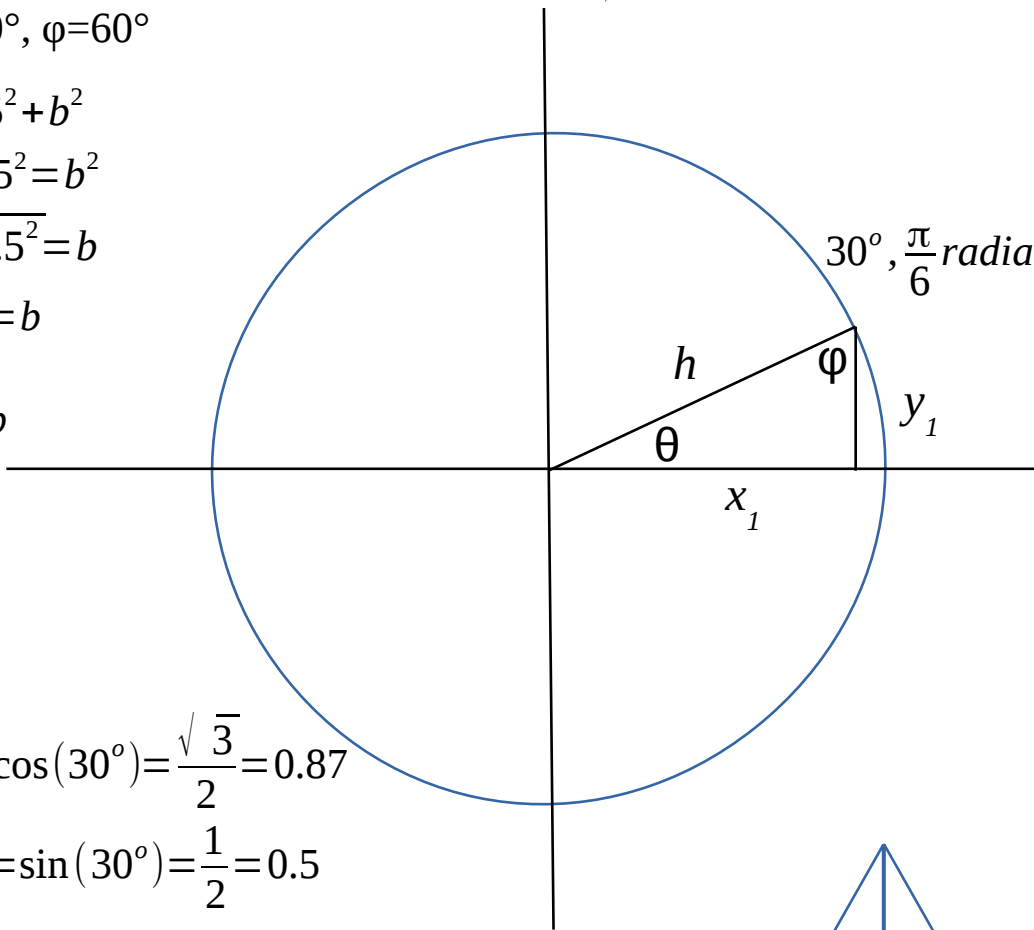The Unit Circle $\rightarrow h = 1$

$\theta = 30°, \varphi=60°$

$1^2=0.5^2+b^2$

$1^2-0.5^2=b^2$

$\sqrt{1^2-0.5^2}=b$

$\sqrt{0.75}=b$

$\sqrt{\dfrac{3}{4}}=b$

$30°, \dfrac{\pi}{6} radians, 33.3\, grads$

h φ

θ y_1

x_1

$b=x_1=\cos(30^o)=\dfrac{\sqrt{3}}{2}=0.87$

$0.5=y_1=\sin(30^o)=\dfrac{1}{2}=0.5$

$30°$ $30°$

1 1

b

$60°$ $90°$ $60°$

0.5 0.5
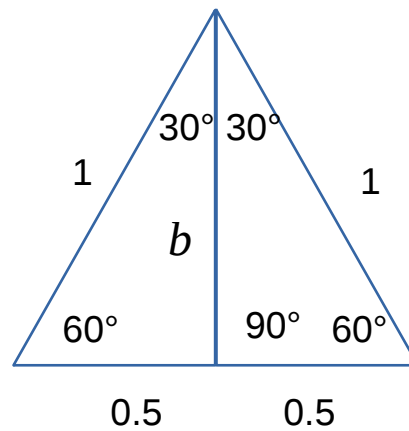
Example: What is $\cos\left(\dfrac{\pi}{4} radians\right)?$

The Unit Circle $\rightarrow h = 1$

$h^2 = x_1^2 + y_1^2 \quad h = 1, \; x_1 = y_1 \; since \; \theta = \varphi$
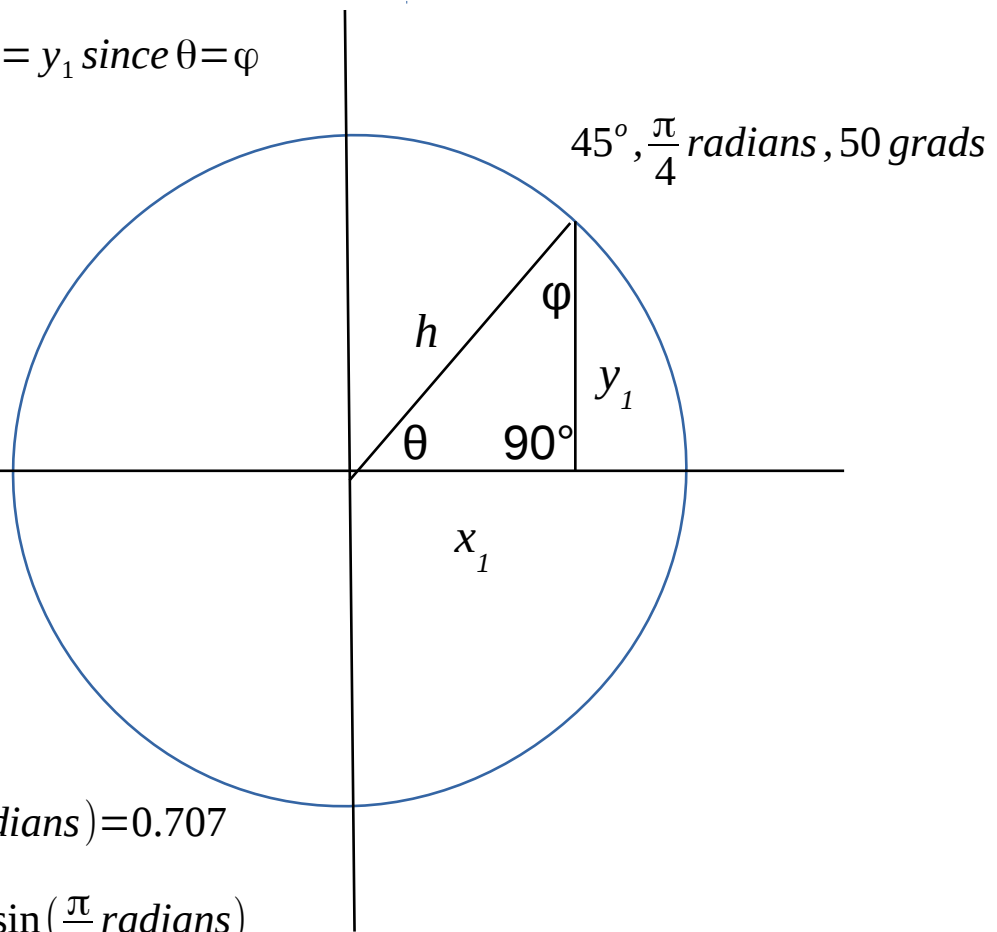
$1^2 = 2 x_1^2$

$1 = 2 x_1^{\,2}$

$\dfrac{1}{2} = x_1^{\,2}$

$\sqrt{\dfrac{1}{2}} = x_1$

$\sqrt{\dfrac{1}{2}} * \dfrac{\sqrt{2}}{\sqrt{2}} = x_1$

$45^\circ, \dfrac{\pi}{4} radians, 50\, grads$

φ

h

y_1

$\theta \qquad 90^\circ$

x_1

$\dfrac{\sqrt{2}}{2} = x_1 = \cos\left(\dfrac{\pi}{4} radians\right) = 0.707$

$Since \; \dfrac{\sqrt{2}}{2} = x_1 = y_1 = \sin\left(\dfrac{\pi}{4} radians\right)$

Example: What is sin(60°)?

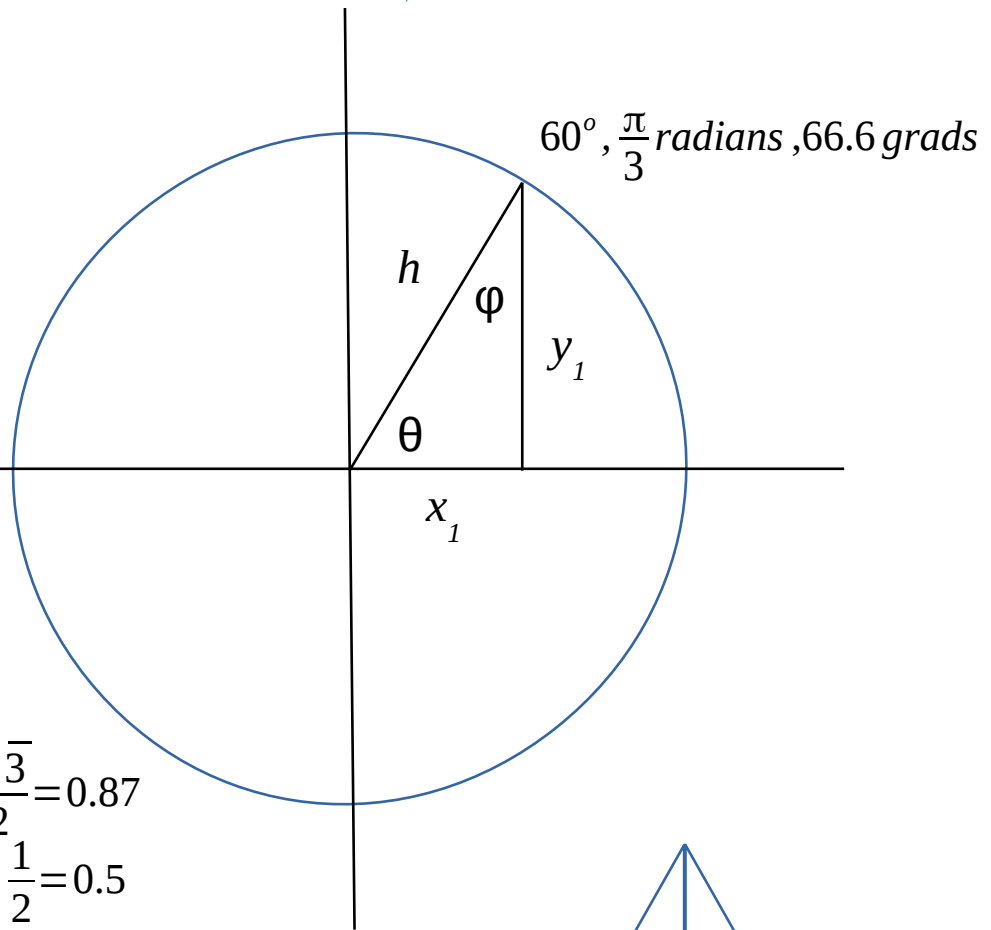The Unit Circle $\rightarrow h = 1$

$\theta = 60°$, $\varphi = 30°$

$1^2 = 0.5^2 + b^2$
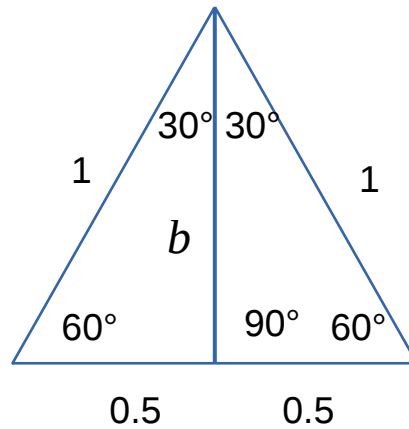
$1^2 - 0.5^2 = b^2$

$\sqrt{1^2 - 0.5^2} = b$

$\sqrt{0.75} = b$

$\sqrt{\dfrac{3}{4}} = b = y_1 = \dfrac{\sqrt{3}}{2}$

$60°, \dfrac{\pi}{3} radians, 66.6\, grads$

h

φ

y_1

θ

x_1

$b = y_1 = \sin(60^o) = \dfrac{\sqrt{3}}{2} = 0.87$

$0.5 = x_1 = \cos(60^o) = \dfrac{1}{2} = 0.5$

30° 30°

1   1

b

60°   90° 60°

0.5   0.5

Example: What is cos(90°)?

The Unit Circle $\rightarrow h = 1$

$\theta = 90°$, $\varphi = 0°$

$y_1 = h = 1$

$x_1 = 0$

$x_1 = \cos(90°) = 0$

$y_1 = \sin(90°) = 1$

y

$(0,1)$ $90°, \frac{\pi}{2} \, radians, 100 \, grads$

φ

h

y_1

θ x

x_1
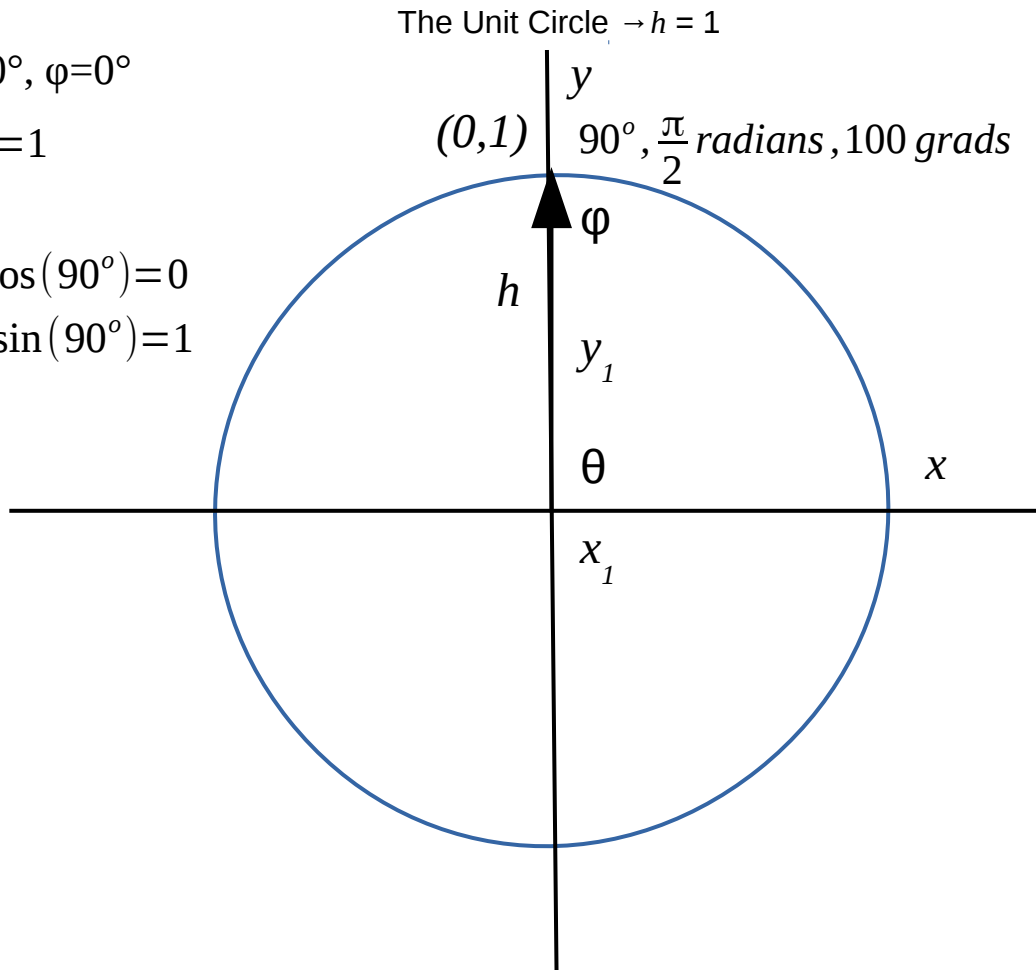
see also
mathsisfun.com/algebra/trigonometry.html